Week 2
MATH 4A
TA: Jerry Luo
jerryluo8@math.ucsb.edu
Website: math.ucsb.edu/~jerryluo8
Office Hours: Monday 9:30-10:30AM, South Hall 6431X; Math Lab hours: Monday 3-5PM, South Hall 1607

3-2.2 Find the value of a for which $v = \begin{bmatrix} -10 \\ 9 \\ -6 \\ a \end{bmatrix}$ is in the span of the set

$$H = span\left\{ \begin{bmatrix} 5 \\ -2 \\ 3 \\ -3 \end{bmatrix}, \begin{bmatrix} 0 \\ -5 \\ 5 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 5 \\ 2 \end{bmatrix} \right\}.$$

Need solution c_1, c_2, c_3 s.t.

$$c_1\begin{bmatrix} 5 \\ -2 \\ 3 \\ -3 \end{bmatrix} + c_2\begin{bmatrix} 0 \\ -5 \\ 5 \\ 4 \end{bmatrix} + c_3\begin{bmatrix} 0 \\ 0 \\ 5 \\ 2 \end{bmatrix} = \begin{bmatrix} -10 \\ 9 \\ -6 \\ a \end{bmatrix}$$

$$\Updownarrow$$

$$\left[\begin{array}{ccc|c} 5 & 0 & 0 & 10 \\ -2 & -5 & 0 & 9 \\ 3 & 5 & 5 & -6 \\ -3 & 4 & 2 & a \end{array}\right] \Leftarrow \text{need to be consistent}$$

$$\Big\downarrow \text{RREF}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & a-4 \end{array}\right] \quad \text{Need } a - 4 = 0 \Leftrightarrow a = 4$$

3-2.3 Find a set of vectors $\{u, v\}$ in $\mathbb{R}^4$ that spans the solution set of

$$\begin{cases} w - x + y - 2z = 0, \\ 3w + 2x - y + z = 0. \end{cases}$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 1 & -2 & 0 \\ 3 & 2 & -1 & 1 & 0 \end{array}\right] \xrightarrow{\text{RREF}} \left[\begin{array}{ccccc} 1 & 0 & \frac{1}{5} & -\frac{3}{5} & 0 \\ 0 & 1 & -\frac{4}{5} & \frac{7}{5} & 0 \end{array}\right]$$

$$w + \frac{1}{5}y - \frac{3}{5}z = 0 \qquad \longrightarrow \qquad w = -\frac{1}{5}y + \frac{3}{5}z$$

$$x - \frac{4}{5}y + \frac{7}{5}z = 0. \qquad \qquad x = \frac{4}{5}y - \frac{7}{5}z.$$

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -\frac{1}{5} - y + \frac{3}{5}z \\ \frac{4}{5}y - \frac{7}{5}z \\ y \\ z \end{bmatrix} = \begin{bmatrix} -\frac{1}{5} \\ \frac{4}{5} \\ 1 \\ 0 \end{bmatrix} y + \begin{bmatrix} \frac{3}{5} \\ -\frac{7}{5} \\ 0 \\ 1 \end{bmatrix}$$

$$u = \begin{bmatrix} -\frac{1}{5} \\ \frac{4}{5} \\ 1 \\ 0 \end{bmatrix} \qquad w = \begin{bmatrix} 3/5 \\ -7/5 \\ 0 \\ 1 \end{bmatrix}$$

3-2.7 Let $a_1 = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$, $a_2 = \begin{bmatrix} h \\ -11 \\ -5 \end{bmatrix}$, and $a_3 = \begin{bmatrix} -10 \\ -14 \\ -5 \end{bmatrix}$

This set will span $\mathbb{R}^3$, unless h is what?

$\{a_1, a_2, a_3\}$ span $\mathbb{R}^3$ ~~unless~~ if and only if lin. indep.

So, we want to know when it's linearly dependent

$\Updownarrow$

there are c_1, c_2 such that $c_1 a_1 + c_2 a_3 = a_2$

$\Updownarrow$

$$\left[\begin{array}{cc|c} 1 & -10 & h \\ 3 & -14 & -11 \\ 1 & -5 & -5 \end{array}\right]$$ consistent (i.e. has a solution)

$\Updownarrow$

RREF...

$\Downarrow$

$\cancel{h \quad 0 \quad -\frac{7}{8}h - \frac{55}{8}}$

$$\begin{bmatrix} 1 & 0 & \frac{-7h-55}{8} \\ 0 & 1 & \frac{-3h-11}{16} \\ 0 & 0 & \frac{-h-25}{16} \end{bmatrix}$$

Need $\frac{-h-25}{16} = 0$

$\Rightarrow h = -25.$

3-2.9 $A = \begin{bmatrix} -3 & 9 & -9 \\ -4 & 14 & -14 \\ 1 & -1 & 1 \end{bmatrix}$. Is it true that $Ax = b$ has a solution for every b?

Method 1: Columns are not linearly independent.

Reason: second column is negative version of third...

Method 2:

$$\begin{bmatrix} -3 & 9 & -9 \\ -4 & 14 & -14 \\ 1 & -1 & 1 \end{bmatrix} \xrightarrow{R_2 - \frac{4}{3}R_1} \begin{bmatrix} -3 & 9 & -9 \\ 0 & 2 & -2 \\ 1 & -1 & 1 \end{bmatrix} \xrightarrow{R_3 + \frac{1}{3}R_1} \begin{bmatrix} -3 & 9 & -9 \\ 0 & 2 & -2 \\ 0 & 2 & -2 \end{bmatrix}$$

$$\xrightarrow{R_2 - R_3} \begin{bmatrix} -3 & 9 & -9 \\ 0 & 2 & -2 \\ 0 & 0 & 0 \end{bmatrix} !!$$

So, it's <u>NOT</u> true that for every b,

$Ax = b$ has a solution